Riemann-Hilbert Problems, families of commuting operators and soliton equations

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We start with Riemann-Hilbert Problems (RHP) with canonical normalization with sewing functions depending on several additional variables. Using Zakharov-Shabat theorem we are able to construct a family of ordinary differential operators for which the solution of the RHP is a common fundamental analytic solution. This family of operators obviously commute. Thus we are able to construct new classes of integrable nonlinear evolution equations.

We illustrate the method with an example of a new types of N-wave interactions. The relevant Lax pairs consist of operators which are both polynomials of order 2, 3 or higher in the spectral parameter λ and take values in simple Lie algebras.